PULSE BROADENING IN MULTIMODE OPTICAL FIBERS WITH LARGE Δn/n. NUMERICAL RESULTS

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Summary

The rms impulse response width of germania doped fibers having power-law profiles at the carrier wavelength is evaluated with the help of a numerical method. For $\Delta n/n \approx 0.02$, the numerical result exceeds by more than one order of magnitude that obtained from an analytical formula based on the assumption that $dn^2/d\lambda$ varies linearly with n^2. Our numerical technique is based on scalar ray optics. It is applicable to any fiber having a large V-number and a smooth profile.

Multimode fibers having relatively large $\Delta \equiv \Delta n/n$ (e.g., $\Delta \gtrsim 0.01$) are attractive because of their high coupling efficiency to LED sources and because of their small sensitivity to microbending loss. Even if long-life monomode injection lasers prove feasible, multimode fibers may remain more attractive than monomode fibers for long-distance high-capacity transmission because of easier cabling and splicing. Links using multimode fibers can easily be upgraded if long-life laser sources become available. Monomode fibers, in contradistinction, are restricted to laser sources.

One key question, however, is whether high data rates can be transmitted through multimode fibers having large Δ. One of the best experimental value reported[1] for the rms impulse response width is $\sigma = 10{,}000\ \Delta^2$nsec/km, for $\Delta = 0.01$. On the other hand, Olshansky and Keck theory[2] suggests that the rms impulse response width does not exceed $150\Delta^2$nsec/km for a power-law profile.

$$n^2(r)/n^2(0) = 1 - 2\Delta(r/a)^{2\kappa} \qquad (1)$$

and a suitable value of the exponent κ. If this theoretical result were applicable to real fibers, gigabit/sec rates would be possible, even for values of Δ as large as 0.02. One obvious explanation for the much higher value observed for σ is that the desired power-law profile has not yet been achieved with sufficient accuracy. We would like to point out another possible explanation: Olshansky and Keck's theory implicitly assumes that $dn^2/d\lambda$ varies linearly with n^2 as the dopant concentration varies. Olshansky and Keck's theory is valid only when the relative group index $\bar{N}$, defined in (6b), is proportional to the relative phase index N, defined in (6a), for the class of materials incorporated in the fiber, or equivalently, when $S \equiv -(\lambda/2)dn^2/d\lambda$ varies linearly with n^2. It is easy to see that when this condition is not fulfilled, power-law profiles do not remain power-law profiles at neighboring wavelengths. In that case it is incorrect to calculate the group velocity by differentiating $n(r,\lambda)$ in (1) with respect to λ. Measurements made recently by Fleming[3] show that $dn^2/d\lambda$ is <u>not</u> a linear function of n^2 (See Fig. 1). The purpose of the present paper is to assess the practical significance of this lack of linearity. This is done by comparing the analytical results in Ref. 2 to exact numerical results. This comparison suggests that the formula given in Ref. 2 is applicable only to fibers with very small values of Δ, typically $\Delta \lesssim 0.005$.

The numerical technique presented in this paper is based on the space-time Hamilton equations.[4] All the information needed to write a working program is supplied here, but the detailed derivations are omitted. This technique gives accurately the rms impulse response width of dispersive fibers having large V-numbers ($V >> 20$) and smooth profiles. About 1 minute of IBM 370 computer time is needed to evaluate the rms impulse response width of a fiber. This is one fifth of the time required by programs based on wave optics. For simplicity, we assume that the source is quasi-monochromatic (e.g., an LED followed by a narrow-band filter). The fiber response for sources with non-zero spectral width is easily obtained by convolving the quasi-monochromatic response evaluated in this letter with the source spectrum.

Let us assume that the Sellmeier-law coefficients A_γ, ℓ_γ, $\gamma = 1, 2, 3$, of the materials incorporated in the fiber (e.g., germania-doped silica) have been measured at the source wavelength. The refractive index n is defined by the Sellmeier law

$$n^2 - 1 = \sum_{\gamma=1}^{3} A_\gamma(1-\pi_\gamma)^{-1}; \quad \pi_\gamma = (\ell_\gamma/\lambda)^2 \qquad (2)$$

We obtain by differentiation with respect to λ

$$S \equiv -(\lambda/2)dn^2/d\lambda = \sum_{\gamma=1}^{3} A_\gamma\pi_\gamma(1-\pi_\gamma)^{-2} \qquad (3)$$

Experimental values of n and S are shown in Fig. 1 for various concentrations of germania or boron oxide in silica. The measurements were made on prism-shaped samples with the minimum deviation method by Fleming.[3] Note that angles of refraction, unlike optical thickness of thin samples, are insensitive to alterations of the sample surface, which may be caused, for example, by oil films or compacting. Note also that the curve $dn^2/d\lambda$

versus n^2 is independent of dopant concentration measurements. Values are interpolated for other concentrations. Whether this interpolation is sufficiently accurate is opened to question. Additional measurements would be highly desirable. However, the information presently available appears to be sufficient to illustrate our point. Because quenching may affect slightly the dependence of $dn^2/d\lambda$ on n^2, it would be desirable to make the measurements directly on fiber samples. However, in spite of recent progress, the measurements on fiber samples that are presently available are too inaccurate to provide the desired information concerning $dn^2/d\lambda$. The profile n(r), however, is most accurately measured on fiber samples.

Let us now describe the ray sampling procedure. We select a normalized azimuthal mode number M and a normalized propagation constant B from the sequences

$$M = 1/A,\ 2/A,\ \ldots \tag{4a}$$

$$B = [(I-1)/I]2\Delta,\ [(I-2)/I]2\Delta,\ \ldots \tag{4b}$$

where $2\Delta \equiv 1 - n_c^2/n_o^2$, $n_o \equiv n(0)$, and n_c is the cladding index. Typical values for A and I in (4) are 50 and 20, respectively. The time of flight of a pulse along the ray trajectory specified by M and B is obtained by solving with the Euler or Runge-Kutta techniques the first order equations

$$dR/dZ = (1-B)^{-1/2}P \tag{5a}$$

$$dP/dZ = (1-B)^{-1/2}(-1/2\ dN/dR + M^2/R^3) \tag{5b}$$

$$dT/dZ = (\bar{N}^2-2\bar{N}+B)/[1-B+(1-\bar{N})(1-B)^{1/2}] \tag{5c}$$

for the three functions R(Z), P(Z) and T(Z). We have defined a relative phase index N and a relative group index $\bar{N}$ by

$$N(R) = 1-n^2(r)/n_o^2 \tag{6a}$$

$$\bar{N}(R) \equiv [N + (S_o-S)n_o^{-2}]/(1 + S_o n_o^{-2}) \tag{6b}$$

where n and S are defined in (2) and (3) respectively, and

$$R \equiv r/a; \qquad S_o \equiv S(0)$$

where a denotes an arbitrary length, perhaps the core radius. (The fiber response is unaffected by a change of scale in the radial direction.) The initial values of R(Z), P(Z), T(Z) are respectively

$$R(0) = R_o;\ dN(R_o)/dR_o = 2M^2/R_o^3 \tag{7a}$$

$$P(0) = [B - N(R_o) - M^2/R_o^2]^{1/2} \tag{7b}$$

$$T(0) = 0 \tag{7c}$$

The derivative dN/dR in (6b) and (7a) can be obtained algebraically in N(R) is given as a simple analytical form such as a power-law profile, or by incrementing R. The integration of (5) terminates after one ray period. Usually, 2000 steps per period are sufficient. The difference between the time of arrival of a pulse along the ray considered and the time of arrival of a pulse along the fiber axis, in nsec/km, is

$$\Delta T = 5{,}000\ T_{final}/Z_{final} \tag{8}$$

The series of values of M and B taken in sequence according to (4) terminates when P(0) in (7b) ceases to be real. For a Lambertian source, the rms impulse response width $\sigma = (\langle\Delta t^2\rangle - \langle\Delta t\rangle^2)^{1/2}$ is obtained by averaging Δt in (8) over all the values of M and B permitted by the condition set up above. For non-Lambertian sources or non-uniform attenuation of the modes, one needs introduce weighting factors in the evaluation of the averages.

The numerical technique just described has been applied to germania doped fibers that have the power-law profile in (1) at the source wavelength, but not necessarily at neighboring wavelengths. We have considered a fiber with 20% germania on axis ($\Delta \approx 0.022$). The cladding is assumed to be made of pure silica. The rms impulse response width of the fiber for a Lambertian quasi-monochromatic source is shown in Fig. 2 by plain lines for three wavelengths of interest as a function of the exponent κ in (1). For comparison, the rms impulse response widths calculated from Ref. 2 are shown by interrupted lines. As one can see, there are large discrepancies between the result obtained from the theory in Ref. 2 and the exact result, particularly when $\lambda > 0.9\mu m$ and $\Delta > 0.01$. For example, when $\lambda = 1.2\mu m$ and $\Delta = 0.0216$, the simplified analytical formula in Ref. 2 predicts that the minimum value of σ is 0.08 nsec/km when the exponent κ is equal to 0.965. For that value of κ, σ is in fact, equal to 2 nsec/km. The minimum value of σ is obtained for $\kappa = 1.05$ and is equal to 0.6 nsec/km.

The analytical formula presented by this author in Ref. 4 can handle arbitrary variations of $dn^2/d\lambda$ as a function of n^2. However, it is restricted to small departures of the profile from a square-law. For $\kappa = 1$ (square-law medium) our analytical result in Ref. 4, shown by black dots in Figs. 1 and 2, agrees with the numerical result to better than three decimal places.

The determination of the optimum profile for a given class of material can be made, using our numerical technique, by successive approximations. For example, for a fiber with 13.5 mole percent germania on axis ($\Delta = 0.014$) and $\lambda = 1.06\mu m$, one finds that the rms impulse response width can be reduced to 52 psec/km by a proper selection of the coefficients of the expansion of N in series of R^2. This near-optimum profile departs very significantly from a power-law profile. The departure of the optimum profile from a power-law profile is even more pronounced when two dopant materials are used.

In conclusion, we have shown that, in order to evaluate the rms impulse response width of a fiber with $\Delta \gtrsim 0.005$, it is essential to measure the variation of $dn^2/d\lambda$ as a function of n^2 at the carrier wavelength, as well as the profile n(r) of the fiber, and to use a theory that takes the actual variation of $dn^2/d\lambda$ into account. The transmission rate capacity of fibers with Δ as large as 0.02 appears to be in the gigabit/sec range, but the index profiles that one must look for are not, in general, power-law profiles.

Acknowledgments

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References

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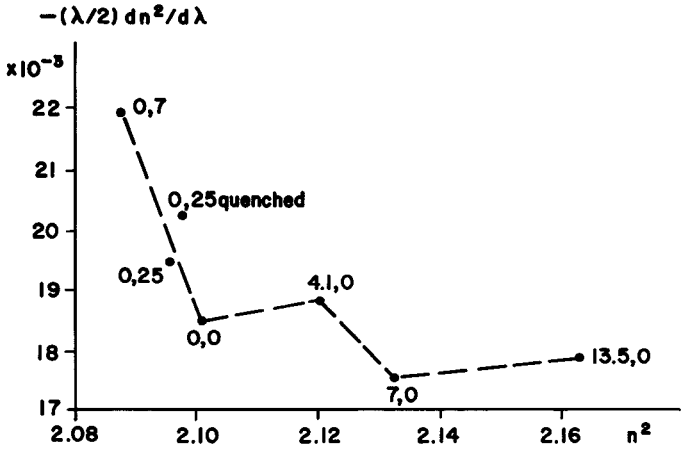

Fig. 1 Variation of $S \equiv -\lambda n(dn/d\lambda)$ as a function of n^2 for various materials at $\lambda = 1.06\mu m$. The numbers near each black point are the concentrations of germania and boron oxide, respectively, in mole percent. S is measured with an accuracy better than 1%. The result in Fig. 2 is obtained from a parabolic interpolation between the values measured for 0, 7 and 13.5% germania.

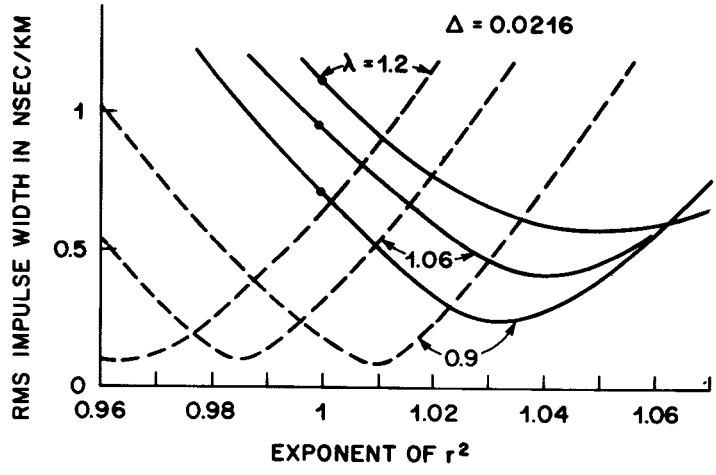

Fig. 2 Variation of the rms impulse response width σ as a function of the exponent κ of r^2 for a germania doped fiber with power-law profile at the carrier wavelength (plain lines). The results from the approximate theory in Ref. 2 are shown as dotted lines. The black dots are from the analytical result in Ref. 4. The germania concentration is assumed to be 20 mole percent on axis, and zero at the cladding.